1. Simplify the following and rearrange to make x the subject of the expression:

(a) $\frac{x}{5} - 3 = 4 + \frac{x}{10}$ (4)

(b) $3\frac{3}{10}x \times 1\frac{4}{5}x = 4$ (4)

a) $\frac{x}{5} - 3 = 4 + \frac{x}{10}$

$\frac{x}{5} - \frac{x}{10} = 7$

$\frac{2x}{10} - \frac{x}{10} = 7$

$\frac{x}{10} = 7$

$x = 70$

b) $3\frac{3}{10}x \times 1\frac{4}{5}x = 4$

$\frac{33}{10}x \times \frac{9}{5}x = 4$

$\frac{297}{15}x^2 = 4$

$x^2 = \frac{60}{297}$

$x = \pm\sqrt{\frac{60}{297}}$

2. (a) John designs a new type of valve seal and offers to sell it to a manufacturer for £25,000.00. The manufacturer likes the design and offers John an initial payment of £8,000.00 plus a royalty of £9.40 for every valve sold with the new seal.

Outline how many valves must be sold for John to achieve his target. (4)

(b) Demonstrate an algebraic method to find three consecutive numbers whose sum is 45. (4)

a) $25,000 - 8000 = 17,000$

$\frac{17000}{9.40} = 1808.51$

1809

b) $(n) + (n+1) + (n+2) = 45$

$3n + 3 = 45$

$3n = 42$

$n = 14$

14, 15, 16

3. During an experiment to test a spring, a mass of 50 grams is suspended from the spring causing it to extend to a total length of 79 mm.

When a mass of 120 grams is suspended from the spring the total spring length is measured to be 98.6 mm. It is initially assumed that the relationship between the spring extension and the added mass is linear.

(a) Draw a graph representing the relationship using the two known data points. (3)

(b) Determine an equation representing the function. (3)

(c) Determine from the graph what the total spring length would be with a mass of 180 grams. (2)

(d) Determine how long the spring is before any mass is added. (2)

| extension mm | load g |
|---|---|
| 79 | 50 |
| 98.6 | 120 |

b) $y = mx + c$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{120 - 50}{98.6 - 79} = \boxed{3.571428} = \frac{25}{7}$$

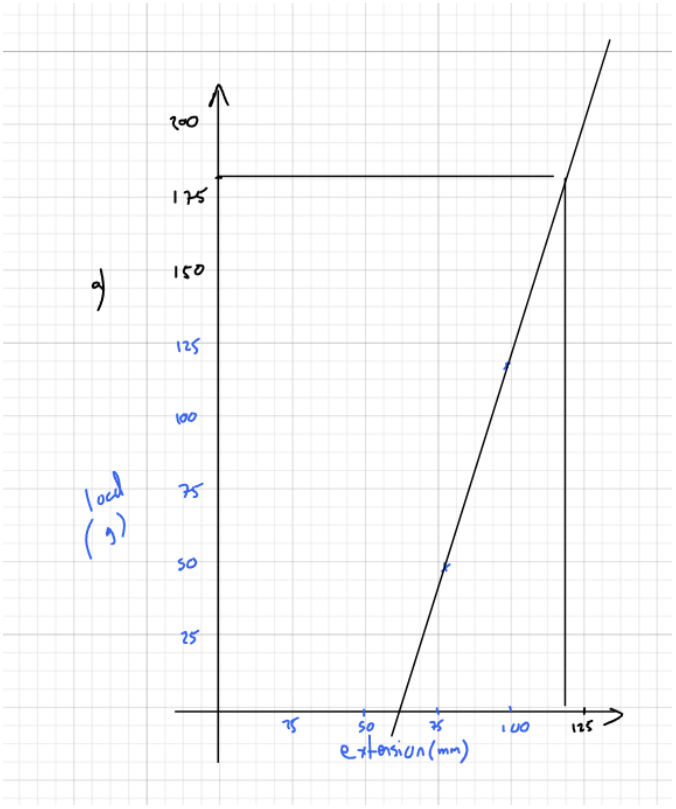
find c when $x = 79$ $y = 50$

$$50 = \frac{25}{7}(79) + c$$

$$-\frac{1625}{7} = c$$

$$-232.1429$$

$$\boxed{y = 3.571428x - 232.1429}$$




c) when mass = 180 g

extension ≈ 120 mm ← from graph

check

$y = 3.571428x - 232.1429$

$y = 3.57142(120) - 232.14$

$= 196.43$ g ✓ ok!

115.5 ← should be from calc

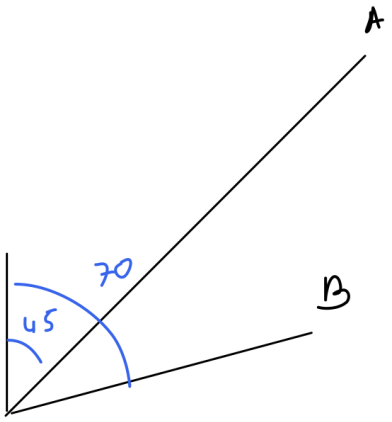
d) $0 = 3.571429x - 232.1429$

$\frac{232.1429}{3.571428} = x$

65 mm

4. Two ships leave a port at the same time. Ship A is on a bearing of 45 degrees travelling at 24 km/h, ship B is on a bearing of 70 degrees travelling at 15km/h.

Calculate how far apart the ships are after 4 hours. (8)




A: $24 \times 4 = 96$ km

B: $15 \times 4 = 60$ km

$70 - 45 = 25°$

$$a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$\sqrt{96^2 + 60^2 - 2(96)(60)\cos 25}$$

$a = \boxed{48.737 \text{ km}}$

5. FIG Q5 shows an equilateral triangle with an inscribed semicircle.

Determine the area of the shaded remainder shown. (8)

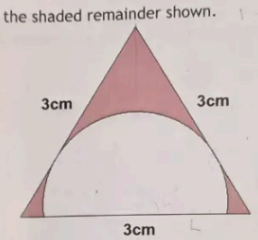
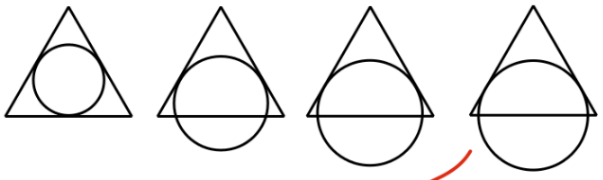
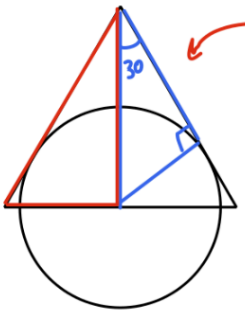



FIG Q5







CAH

$\cos 30 = \frac{y}{3}$

$3\cos 30 = y$

$\frac{3\sqrt{3}}{2} = y = 2.598$

$2.598 = \frac{3\sqrt{3}}{2}$


SOH

$\sin 30 = \frac{x}{\frac{3\sqrt{3}}{2}}$

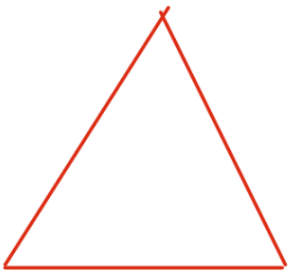
$\frac{3\sqrt{3}}{2}\sin 30 = x$

$1.29903 = \frac{3\sqrt{3}}{4} = x$

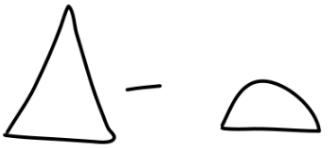
$\text{Area} = \frac{\pi r^2}{2}$

$= \frac{\pi (1.29903)^2}{2}$

$= 2.65068 \text{ cm}^2$



$\text{Area} = \frac{b \times h}{2} = \frac{3 \times \frac{3\sqrt{3}}{2}}{2} = \frac{9\sqrt{3}}{4} = 3.8971$



$3.8971 - 2.65068 = 1.2464 \text{ cm}^2$

6. An oil storage tank is constructed from a round steel cylinder with an outer radius of 360 mm and flat circular ends giving it an overall length of 1.5 m, the steel has a wall thickness of 10 mm.

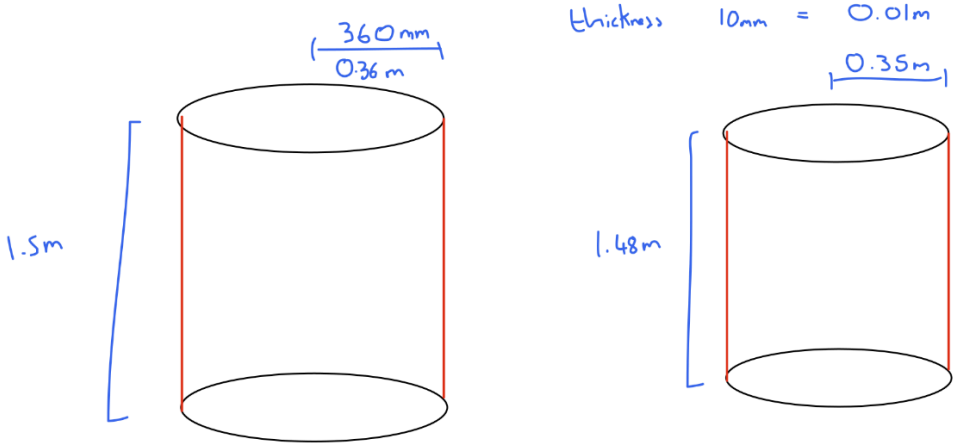
Determine EACH of the following:

(a) the mass of oil in the tank when full; (4)

(b) the mass of the tank when it is empty. (4)

*Note: Density of oil = 900 kg/m³, Relative density of steel = 7.8*

Thickness 10mm = 0.01m




$Vol = \pi r^2 h \quad - \quad \pi r^2 h$

$\pi(0.36)^2(1.5) \quad - \quad \pi(0.35)^2(1.48)$

$0.6107256 \quad - \quad 0.5695707 \quad = \quad \boxed{0.04115485\ m^3}$

6. An oil storage tank is constructed from a round steel cylinder with an outer radius of 360 mm and flat circular ends giving it an overall length of 1.5 m, the steel has a wall thickness of 10 mm.

Determine EACH of the following:

(a) the mass of oil in the tank when full; (4)

(b) the mass of the tank when it is empty. (4)

*Note: Density of oil = 900 kg/m³, Relative density of steel = 7.8*

a) 


$m = d \times v$

$900 \times 0.5695707 = \boxed{512.61\ kg}$

b) $0.04115485\ m^3 \times 7800 = \boxed{321.01\ kg}$

7. (a) Explain what is meant by Hooke's Law. (2)

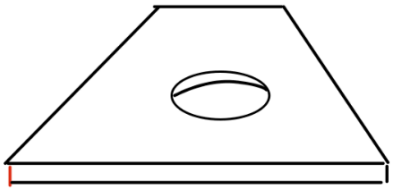
(b) A round hole, 50 mm diameter, is to be punched through a metal plate 2 mm thick. The metal has a shear strength of 120 MPa.

Determine EACH of the following:

(i) the minimum force required to punch the hole; (3)

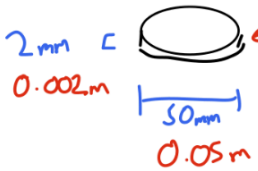
(ii) the maximum compressive stress on the punch during this process. (3)

b)



$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = 120 \text{ MPa} = 120 \times 10^6 \text{ N/m}^2$$




$$\text{Area} = \pi dh = \pi (0.05)(0.002) = 3.1459 \times 10^{-4} \text{ m}^2$$

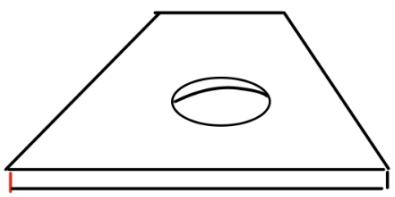
$$\text{Stress} \times \text{Area} = \text{Force}$$

$$120 \times 10^6 \times 3.1459 \times 10^{-4} =$$

$$\boxed{37699.11 \text{ N}}$$

3.7 t

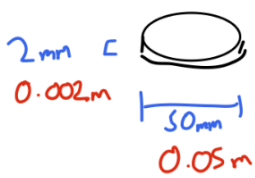
b)



$$\text{Stress} = \frac{F}{A} = \frac{37699.11}{\pi (0.025)^2}$$

$$= 19200000$$

$$\boxed{19.2 \times 10^6 \text{ N/m}^2}$$



8. A pump discharges sea water vertically through a 60 mm diameter pipe to a height of 9 m at a constant velocity of 5 m/s.

Determine EACH of the following:

(a) the volumetric flow rate of water; (3)

(b) the pressure at the pump exit; (3)

(c) the power of the pump; (2)

(d) the motor power to drive the pump if the mechanical efficiency is 86%. (2)

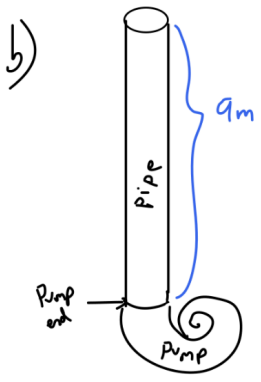
*Note: Density of seawater = 1025 kg/m³.*

a) vol flow rate = Cross section × Vel

$(0.03)^2 \pi \times 5 = 0.014137\ m^3/sec$

14.137 litres/sec

b)




$P = \rho g h$

$= 1025 \times 9.81 \times 9$

90,497.25 ($P_a$)

0.90497 bar

8. A pump discharges sea water vertically through a 60 mm diameter pipe to a height of 9 m at a constant velocity of 5 m/s.

Determine EACH of the following:

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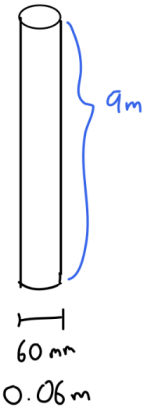
(c) the power of the pump; (2)

(d) the motor power to drive the pump if the mechanical efficiency is 86%. (2)

*Note: Density of seawater = 1025 kg/m³.*

c)




vel = 5

$P = \frac{mgh}{t}$

$vol = \pi r^2 h$

$= \pi (0.03)^2 (9)$

$= 0.02544\,69\ m^3$


$m = d \times v$

$1025 \times 0.02544\,69$

$= 26.083\ kg$

$P = 26.083 \times 9.81 \times 5$

$(1279.37)$ Watt

d)

motor power $\frac{1279.37}{0.86} = \boxed{1487.65 \text{ Watts}}$

9. A piston has a mass of 30 kg. Its velocity increases uniformly in part of its stroke from 1.0 m/s to 2.4 m/s in 0.4 seconds.

(a) Calculate the accelerating force required. (2)

(b) The piston is now uniformly retarded at 6 m/s² from 2.4 m/s until the mass just comes to rest.

Determine for the retardation period EACH of the following:

(i) the time taken to come to rest; (4)

(ii) the distance travelled. (2)

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \left(\frac{u+v}{2}\right)t$

9)

$s$

$u = 1$ m/s

$v = 2.4$ m/s

$a = x$

$t = 0.4$ sec

$v = u + at$

$2.4 = 1 + x(0.4)$

$\frac{1.4}{0.4} = x$

accel $= 3.5$ m/s²

$F = ma$

$F = 30 \times 3.5$

$= \boxed{105\text{ N}}$

9. A piston has a mass of 30 kg. Its velocity increases uniformly in part of its stroke from 1.0 m/s to 2.4 m/s in 0.4 seconds.

(a) Calculate the accelerating force required. (2)

(b) The piston is now uniformly retarded at 6 m/s² from 2.4 m/s until the mass just comes to rest.

Determine for the retardation period EACH of the following:

(i) the time taken to come to rest; (4)

(ii) the distance travelled. (2)

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \left(\frac{u+v}{2}\right)t$

b) i)

$s$

$u = 2.4$

$v = 0$

$a = -6$ m/s²

$t = x$

$v = u + at$

$0 = 2.4 - 6x$

$\frac{2.4}{6} = x$

$\boxed{0.4\text{ sec}}$

ii)

$s$

$u = 2.4$

$v = 0$

$a = -6$ m/s²

$t = 0.4$ sec

$s = \left(\frac{u+v}{2}\right)t$

$\left(\frac{2.4+0}{2}\right)0.4$

$(1.2)\,0.4$

$\boxed{0.48\text{ m}}$

10. For the shape shown in Fig Q10 determine the position of the centroid in the x and y planes. (8)

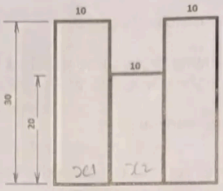
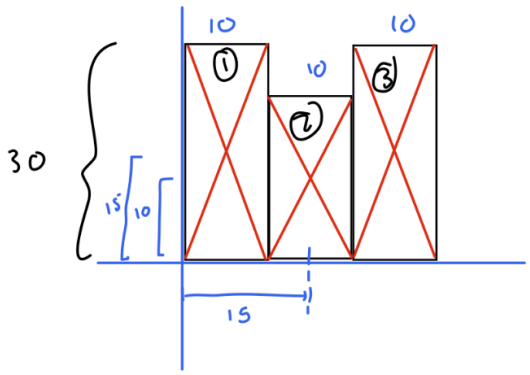
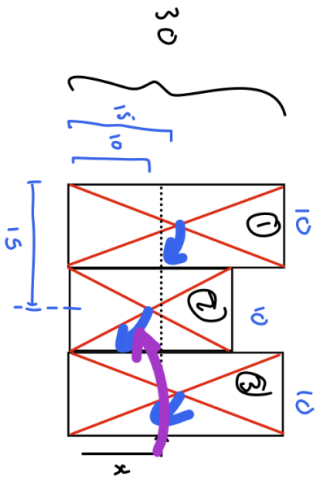



Fig Q10




only need to do vertical balancing as the horizontal is symmetrical




taking moments about bottom left hand corner

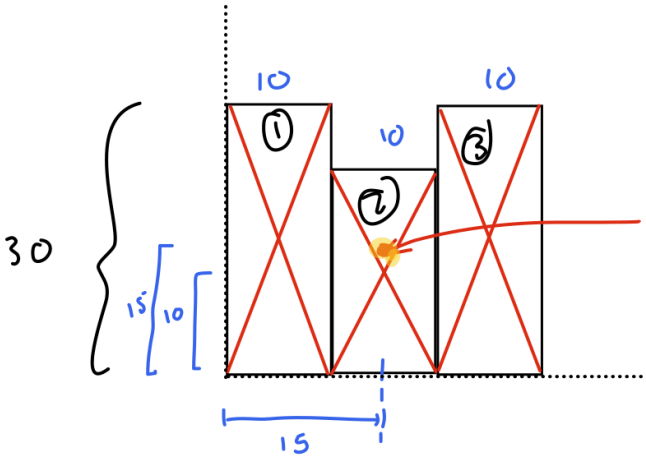
| Name | Area | Distance | Moment | Dir |
|---|---|---|---|---|
| ① | 300 | 15 | 4500 | C |
| ② | 200 | 10 | 2000 | C |
| ③ | 300 | 15 | 4500 | C |
| Fulcrum | 800 | x | 800x | A |

sum of clockwise moments = sum of anticlockwise moments

$$4500 + 2000 + 4500 = 800x$$

$11000 = 800$

$13.75 = x$




centroid location at (15, 13.75) units from the bottom left hand corner

11. Engineers have to consider the effects of friction in most situations.

(a) Briefly describe EACH of the following:

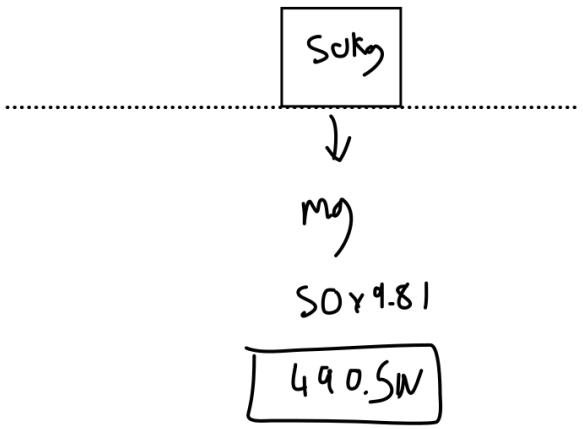
(i) an example where friction is a problem and its effect; (2)

(ii) an example where friction is useful and its effect. (2)

(b) A box has a mass of 50kg and requires a horizontal force of 85N to slide it along the deck.

Determine the dynamic coefficient of friction. (4)

b)




$Fric = \mu N$

$85 = \mu (490.5)$

$\frac{85}{490.5} =$

$0.17329 = \mu$

12. A rotor has a diameter of 0.8 m and rotates at 460 rad/s.

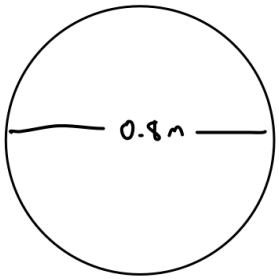
Determine EACH of the following:

(a) the linear velocity of a point on the rim in m/s; (3)

(b) the rotational speed of the flywheel in rev/min; (3)

(c) the number of radians turned when the flywheel rotates through 325°. (2)

a)




linear vel = Radius x Rad/sec

= 0.4 x 460

= 184 m/s

12. A rotor has a diameter of 0.8 m and rotates at 460 rad/s.

Determine EACH of the following:

(a) the linear velocity of a point on the rim in m/s; (3)

(b) the rotational speed of the flywheel in rev/min; (3)

(c) the number of radians turned when the flywheel rotates through 325°. (2)

| Deg | Rev | Rad |
|---|---|---|
| 360 | 1 | 2π |

÷360 ÷2π ×2π

b) $\frac{460 \text{ rad}}{\text{sec}}$ ÷2π ×60 = 4392.67 $\frac{\text{Rev}}{\text{min}}$

c) 325 ÷ 360 ×2π = 5.6723 Rad